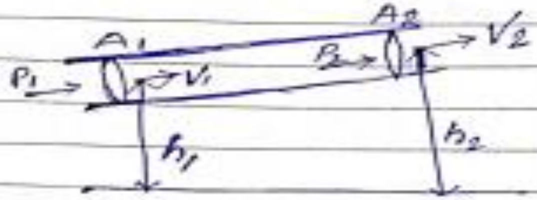
BERNOULLI'S THEOREM

According to Bernoulli's Theorem the sum of energies possessed by a flowing liquid at any point is constant provided the flow is stream-line.



Consider an incompressible liquid in a pipe line which motion is stream-line.
Let the pressures of the liquid at the cross-sections A_1 and A_2 be P_1 and P_2
Let area of cross section at $A_1 = a_1$ and at $A_2 = a_2$. Velocity of flow at A_1 is v_1 & at A_2 is v_2.
Work done per second on the liquid entering A_1

$$W_1 = P_1 a_1 v_1$$

and work done per second by the liquid leaving A_2

$$W_2 = P_2 a_2 v_2$$

$\therefore$ Net work done on the liquid

$$W = W_1 - W_2 = P_1 a_1 v_1 - P_2 a_2 v_2$$

$\because \; a_1 v_1 = a_2 v_2$

$$\therefore W = (P_1 - P_2) a_1 v_1$$

This work done on the liquid contributes for the changes in kinetic energy and gravitational energy.

Change in ~~gravitational~~ potential energy $E_1 = (a_1 v_1) \rho g (h_2 - h_1)$

where ρ is density of the liquid.

Change in kinetic energy $\rightarrow E_2 = \frac{1}{2}(a_1 v_1)\rho(v_2^2 - v_1^2)$

$\therefore \; W = E_1 + E_2$

or $(P_1 - P_2) a_1 v_1 = (a_1 v_1)\rho g (h_2 - h_1) + \frac{1}{2}(a_1 v_1)\rho(v_2^2 - v_1^2)$

or $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$

or $\frac{v_1^2}{2} + g h_1 + \frac{P_1}{\rho} = \frac{v_2^2}{2} + g h_2 + \frac{P_2}{\rho}$

or $\frac{v_1^2}{2} + g h_1 + \frac{P_1}{\rho} - \frac{v_2^2}{2} - g h_2 - \frac{P_2}{\rho} = 0$

or $\frac{(v_1^2 - v_2^2)}{2} + g(h_1 - h_2) + \frac{(P_1 - P_2)}{\rho} = 0$

or $\boxed{\frac{v^2}{2} + g h + \frac{P}{\rho} = \text{constant}}$ — (1)

this equ^n represents Bernoulli's equation.

Dividing equ^n (1) by g we get

$$\left(\frac{v^2}{2g}\right) + h + \left(\frac{P}{\rho g}\right) = \text{constant}$$

~~Hence~~ $\frac{v^2}{2g}$ represents the velocity head

$h \rightarrow$ represents gravitational head / potential head

and $\frac{P}{\rho g} \rightarrow$ represents pressure head.

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B.Sc-I PHYSICS(H)
Paper-1 GENERAL PROPERTIES OF MATER
BY- Dr. Shashi Shekhar Upadhyay

POISSON's RATIO

whenever a body is subjected to a force in a particular direction, there is change in dimensions of the body in the other two perpendicular directions. This is called lateral strain.

Let α be the longitudinal strain per unit stress and β the lateral strain per unit stress, within the elastic limit

$$\beta \propto \alpha$$

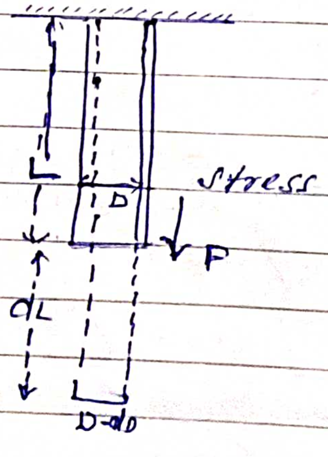
or $\beta = \sigma \alpha$

$$\sigma = \frac{\beta}{\alpha} \quad \text{—} \quad (1)$$

where σ is a constant called Poisson's ratio.

Therefore, Poisson's ratio is defined as the ratio of lateral strain per unit stress to the longitudinal strain per unit stress.

Consider a wire of length L and diameter D. fixed at one end and force is applied at the other end then the length of wire increased and the diameter of wire decreased.



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Suppose, increase in length is dL, and decrease in diameter is dD

$$\therefore \sigma = \frac{\beta}{\alpha} = -\frac{dD/D}{dL/L}$$

or $$\sigma = -\left(\frac{dD}{dL}\right)\left(\frac{L}{D}\right) \quad \text{— (2)}$$

−ve sign indicates that the increase in length there is decrease in diameter.

σ has no unit because it is ratio of numbers.

Value of poison's ratio:– For most of the substances, the value of σ varies with between 0.2 and 0.4.

If the volume of the wire remains unchanged after the force has be applied then

$$V = \left(\frac{\pi D^2}{4}\right)L$$ ∵ volume of wire $= \pi r^2 L$ Here $r = \frac{D}{2}$

Differentiating above eqn w.r.to diameter

$$dV = \frac{\pi}{4}\left[D^2 dL + 2LD.dD\right]$$

If $dV = 0$

Then $D^2 dL + 2LD\,dD = 0$

or, $$\frac{dD}{dL} \times \frac{L}{D} = -\frac{1}{2}.$$

But from eqn (2)

$$\sigma = -\left(\frac{dD}{dL}\right)\left(\frac{L}{D}\right)$$

$$\therefore \sigma = -\left(-\frac{1}{2}\right)$$

or $$\boxed{\sigma = \frac{1}{2}} \quad \text{— (3)}$$

This is the maximum value of Poison's ratio i.e 0.5

—x—